

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/13

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

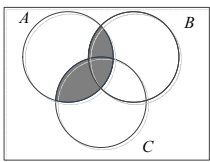
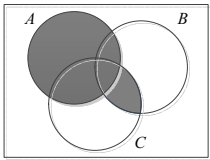
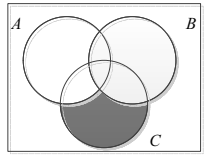
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

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Abbreviations

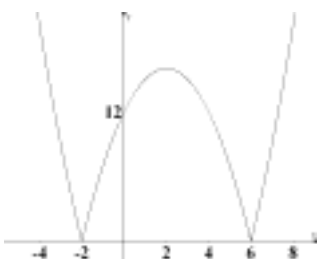
Awrt	answers which round to
Cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	(i)		B1	
	(ii)		B1	
	(iii)		B1	
2	$\cos\left(3x - \frac{\pi}{4}\right) = (\pm)\frac{1}{\sqrt{2}} \text{ oe}$ $3x - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$ $x = \left(-\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{3\pi}{4} + \frac{\pi}{4}\right) \div 3 \text{ oe}$ $x = 0 \text{ and } \frac{\pi}{6} \text{ (or 0 and 0.524)}$ $x = \frac{\pi}{3} \text{ (or 1.05)}$			
			M1	division by 2 and square root
			DM1	correct order of operations in order to obtain a solution
			A2/1/0	A2 for 3 solutions and no extras in the range A1 for 2 solutions A0 for one solution or no solutions

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3	(a)	$\begin{pmatrix} 12 & 16 & 4 \\ 30 & 32 & 10 \end{pmatrix}$	B2,1,0	B2 for 6 elements correct, B1 for 5 elements correct
	(b)	$\begin{pmatrix} 28 & -24 \\ -8 & 76 \end{pmatrix} = m \begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix} + n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $-24 = 6m$ or $-8 = 2m$ giving $m = -4$ $28 = 4m + n$ or $76 = -8m + n$ $n = 44$	B2,1,0 B1 M1 A1	B2 for 4 correct elements in $\mathbf{X}^2$ B1 for 3 correct elements in $\mathbf{X}^2$ For $m = -4$ using correct I complete method to obtain n
	(c)	$a^2 - 6 = 0$ so $a = \pm\sqrt{6}$	B2,1,0	B2 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$, with no incorrect statements seen or B1 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$ seen or B1 for $a = \sqrt{6}$ and no incorrect working
4	(i)	$\frac{1}{2}(4\sqrt{3}+1) \times BC = \frac{47}{2}$ $BC = \frac{47}{(4\sqrt{3}+1)} \times \frac{(4\sqrt{3}-1)}{(4\sqrt{3}-1)}$ $BC = 4\sqrt{3}-1$ Alternative method $\frac{1}{2}(4\sqrt{3}+1) \times BC = \frac{47}{2}$ $(4\sqrt{3}+1)(a\sqrt{3}+b) = 47$ Leading to $12a + b = 47$ and $a + 4b = 0$ Solution of simultaneous equations $BC = 4\sqrt{3}-1$	B1 M1 A1	correct use of the area correct rationalisation Dependent on all method being seen
		$(4\sqrt{3}+1)^2 + (4\sqrt{3}-1)^2$ $= (48 + 8\sqrt{3} + 1) + (48 - 8\sqrt{3} + 1)$ $AC^2 = 98$ $AC = 7\sqrt{2} \text{ or } p = 7$	B1 M1 A1	Dependent on all method seen including solution of simultaneous equations
	(ii)		B1FT	6 correct FT terms seen
			B1cao	98 and $7\sqrt{2}$ or 98 and $p = 7$

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5	When $x = \frac{\pi}{4}, y = 2$	B1	$y = 2$
	$\frac{dy}{dx} = 5\sec^2 x$	B1	$5\sec^2 x$
	When $x = \frac{\pi}{4}, \frac{dy}{dx} = 10$	B1	10 from differentiation
	Equation of normal $y - 2 = -\frac{1}{10}\left(x - \frac{\pi}{4}\right)$	M1	$y - \text{their } 2 = -\frac{1}{\text{their } 10}\left(x - \frac{\pi}{4}\right)$
	$10y + x - 20 - \frac{\pi}{4} = 0$ or $10y + x - 20.8 = 0$ oe	A1	allow unsimplified
6 (i)		B1	shape
		B1	intercepts on x-axis
		B1	intercept on y-axis for a curve with a maximum and two arms
		M1	$(2, \pm 16)$ seen or $(2, k)$ where $k > 0$
		A1	$(2, 16)$ or $x = 2$ and $y = 16$ only
(ii)	$(2, 16)$		
(iii)	$k = 0$	B1	
	$k > 16$	B1	

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7	$\frac{dy}{dx} = 2 \sin 3x \quad (+c)$ $4\sqrt{3} = 2 \frac{\sqrt{3}}{2} + c$ $\frac{dy}{dx} = 2 \sin 3x + 3\sqrt{3}$ $y = -\frac{2}{3} \cos 3x + 3\sqrt{3}x \quad (+d)$ $-\frac{1}{3} = -\frac{2}{3} \cos \frac{\pi}{3} + 3\sqrt{3} \left(\frac{\pi}{9} \right) + d$ $y = -\frac{2}{3} \cos 3x + 3\sqrt{3}x - \frac{\sqrt{3}}{3} \pi$	B1 M1 A1 B1FT M1 A1	$2 \sin 3x$ finding constant using $\frac{dy}{dx} = k \sin 3x + c$ making use of $\frac{dy}{dx} = 4\sqrt{3}$ and $x = \frac{\pi}{9}$ Allow with $c = 5.20$ or $\sqrt{27}$ FT integration of <i>their</i> $k \sin 3x$ finding constant d for $k \cos 3x + cx + d$ Allow $y = -0.667 \cos 3x + 5.20x - 0.577\pi$ or better
8 (a)	$(2 + kx)^8 = 256 + 1024kx + 1792k^2x^2 + 1792k^3x^3$ $k = \frac{1}{4}$ $p = 112$ $q = 28$	B1 B1FT B1FT	 FT 1792 multiplied by <i>their</i> k^2 FT 1792 multiplied by <i>their</i> k^3
(b)	${}^9C_3 x^6 \left(-\frac{2}{x^2} \right)^3$ $84x^6 \left(-\frac{8}{x^6} \right) \text{ leading to } -672$	M1 DM1 A1	correct term seen Term selected and 2^3 and 9C_3 correctly evaluated

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9	(a) (i)	Number of arrangements with Maths books as one item = $4!$ or $4 \times 3!$	M1	$4!(\times 2)$ or $4 \times 3!(\times 2)$ oe
		or Maths books can be arranged 2! ways and History 3! ways = $2! \times 3!$		$2! \times 3!(\times 4)$ or $2 \times 3!(\times 4)$ oe
		$2 \times 4!$ or $2 \times 4 \times 3!$ or $4 \times 2 \times 3! = 48$	A1	A1 for 48
	(ii)	$5! - 48$ or $6 \times 2 \times 3!$	M1	$5!$ – <i>their</i> answer to (i)
		72	A1	or for $6 \times 2 \times 3$
	(b) (i)	3003	B1	
		(ii) $3003 - 6 - 135$	M1	<i>their</i> answer to (i) – $6 - {}^6C_4 \times 9$
		2862	B1	135 subtracted
		or	A1	
		2M 3W = 720	M1	complete correct method using 4 cases,
		3M 2W = 1260		may be implied by working. Must have
		4M 1W = 756		at least one correct
		5M = 126	B1	any 3 correct
		2862	A1	

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10	(i)	$10^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos ABC$ or $\sin\left(\frac{ABC}{2}\right) = \frac{5}{6}$ or $ABC = \pi - \sin^{-1} \frac{10\sqrt{11}}{36}$ $ABC = 1.9702$	M1	correct cosine rule statement or correct statement for $\sin \frac{ABC}{2}$ or equating areas oe
			A1	1.9702 or better
	(ii)	$XY = 2$ Arc length $6\left(\frac{\pi - 1.970}{2}\right)$ oe	B1	for XY (may be implied by later work, allow on diagram)
			B1	correct arc length (unsimplified)
		Perimeter = $2 + 2\left(6\left(\frac{\pi - 1.970}{2}\right)\right)$ = 9.03	M1	<i>their</i> $2 + 2 \times 6 \times$ <i>their</i> angle C
			A1	
	(iii)	$\left(\frac{1}{2} \times 6^2 \left(\frac{\pi - 1.970}{2}\right) - \frac{1}{2} \times 5 \times \sqrt{11}\right) \times 2$ = 4.50 or 4.51 or better	M1	sector area using <i>their</i> C
			M1	area of $\triangle ABM$ where M is the midpoint of AC , or ($\triangle ABY$ and BXY) or $\triangle ABC$
			A1	Answers to 3sf or better

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11	$x^2 - 2x - 3 = 0$ or $y^2 - 6y + 5 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable
	leading to (3, 5) and (-1, 1)	A1,A1	A1 for each 'pair' from a correct quadratic equation, correctly obtained.
	Midpoint (1, 3)	B1cao	midpoint
	(Gradient - 1)	M1	perpendicular bisector, must be using <i>their</i> perpendicular gradient and <i>their</i> midpoint
	Perpendicular bisector $y = 4 - x$	M1	substitution and simplification to obtain a three term quadratic equation in one variable.
	Meets the curve again if $x^2 + 10x - 15 = 0$ or $y^2 - 18y + 41 = 0$	M1	
	leading to $x = -5 \pm 2\sqrt{10}$, $y = 9 \mp 2\sqrt{10}$	A1,A1	A1 for each 'pair'
	$CD^2 = (4\sqrt{10})^2 + (4\sqrt{10})^2$	M1	Pythagoras using <i>their</i> coordinates from solution of second quadratic. $(x_1 - x_2)^2 + (y_1 - y_2)^2$ must be seen if not using correct coordinates.
	$CD = 8\sqrt{5}$	A1	A1 for $8\sqrt{5}$ from $\sqrt{320}$ and all correct so far.

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12	(a)	$2^{2x-1} \times 2^{2(x+y)} = 2^7$ and $\frac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$	M1	expressing 4^{x+y} , 128 as powers of 2 and 9^{2y-x} , 27^{y-4} as powers of 3
		$2x - 1 + 2(x + y) = 7$ oe	A1	Correct equation from correct working
		$2(2y - x) = 3(y - 4)$ oe	A1	Correct equation from correct working
		leading to $x = 4$, $y = -4$	A1	for both
		<u>Example of Alternative method</u> Method mark as above $2x - 1 + 2(x + y) = 7$	M1	As before
	(b)	leading to $y = \frac{(8 - 4x)}{2}$	A1	One of the correct equations in x and y
		Correctly substituted in $\frac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$		
		Leading to $2\left(\frac{2(8 - 4x)}{2} - x\right) = 3\left(\frac{(8 - 4x)}{2} - 4\right)$	A1	Correct, unsimplified, equation in x or y only
		Leading to $x = 4$ and $y = -4$	A1	Both answers
		$(2(5^z) - 1)(5^z + 1) = 0$	M1	solution of quadratic
		leading to $2.5^z = 1$ ($5^z = -1$)	A1	correct solution
		$5^z = 0.5$	DM1	correct attempt to solve $2.5^z = k$, where k is positive
		$z = \frac{\log 0.5}{\log 5}$ or $z = -0.431$ or better	A1	must have one solution only